Article

The Nature & Sensitivity of Chaos

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ABSTRACT

In this paper, the nature and sensitivity of chaos will be illustrated. Failure to appreciate the generative nature of chaos has led to it being one of the last scientific frontiers to be discovered, over fifty years after relativity and quantum theory. Far from being the nemesis of order, or the primal ooze in which order is imposed, chaos is genesis of new form. Most complex systems arise from the mutual interaction between chaos and order, through bifurcation. The eternal religious war of light and dark is very much the battle of chaos as the dark 'force' and order as the principle of light.

Key Words: chaos, nature, sensitivity, order, new form, genesis, religious war.

The Mythology of Chaos

Chaos Gk. kaos abyss – to 'yawn'or 'gape'

In the Britannica Dictionary chaos is 'a condition of utter disorder or confusion as the unformed primal state of the universe' citing either utter disorder and confusion or an unfathomable abyss as definitive. The Concise Oxford speaks of 'formless void or great deep of primordial matter, this personified as the oldest of the Gods, utter confusion'. The Grollier Encyclopedia notes that in Greek mythology, Chaos was the unorganized state, or void, from which all things arose. Proceeding from time, Chaos eventually formed a huge egg from which there issued Heaven, Earth, and Eros (love). According to Hesiod's Theogeny, Chaos preceded the origin not only of the world, but also of the gods. In Hebrew myth tohu wabohu is the universe without form and void, as in Genesis 1:2:

And the earth was without form, and void;
and darkness was upon the face of the deep.

Barbara Walker likens chaos to the undifferentiated raw elements occupying the womb of the world-goddess between destruction and recreation of the universe.

The eternal religious war of light and dark is very much the battle of chaos as the dark 'force' and order as the principle of light. This is enacted in diverse myths of origin. In Babylon, Tiamat the feminine primal abyss and ancient mother is overthrown by Marduk the youthful male slayer of chaos, in the name of civic, and world order. The same theme extends to classic male combat myth in the cosmic Zoroastrian war of dark and light which became in Jewish and later Christian thought the battle between

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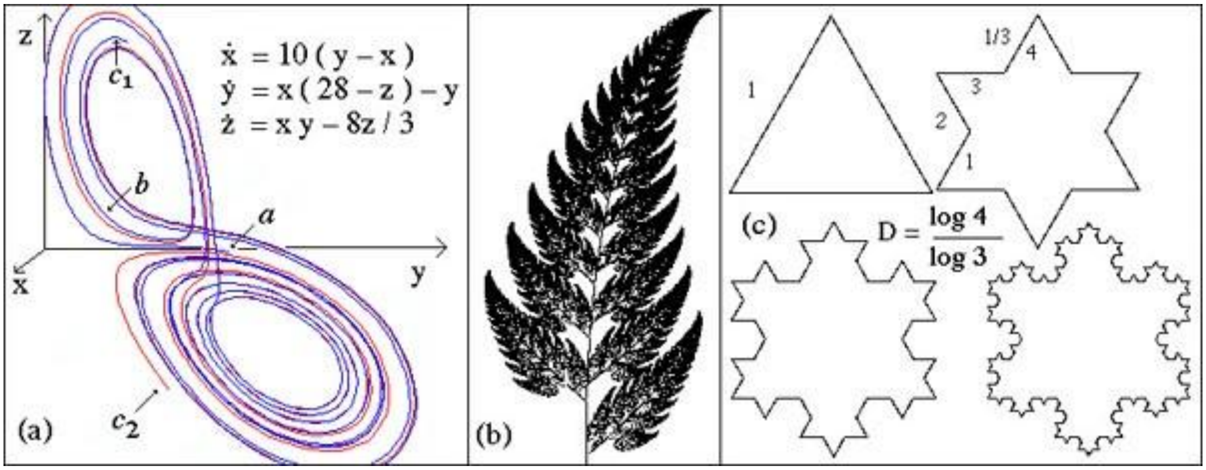
God and Satan which leads to Armageddon and the unveiling tumult of apocalypse. This opposition between chaos and order is a fundamental misunderstanding of the natural condition.

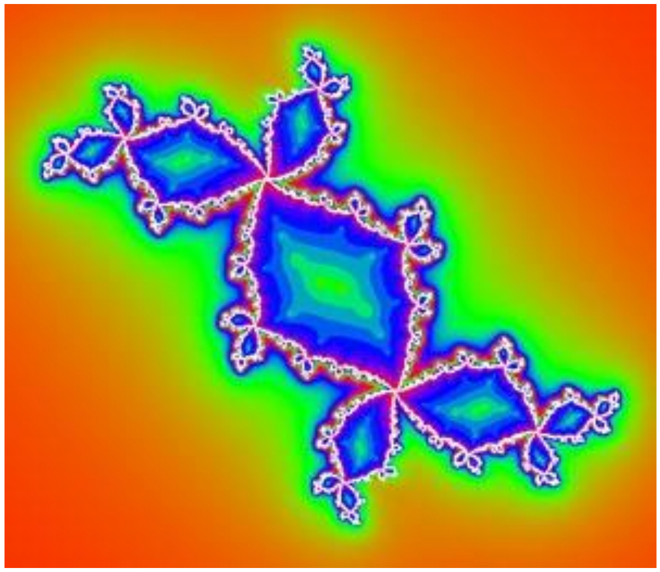


(a) Sensitive dependence on initial conditions is illustrated by the Lorenz flow simulating turbulent weather. Trajectories in the flow starting arbitrarily close at a have exponentially separated by b and are in distinct parts of the flow by c1,2. Computational unpredictability follows from the incapacity of any numerical model to approximate the real flow over increasing time, because of such exponentiating divergence. (b) The fern leaf set is an example of a fractal generated by a simple recursive transformation. (c) Non-integer fractal dimension D of the Koch flake. If we subdivide a line of dimension 1 into pieces we get 3^1 units of 1/3 the length . If we have a planar region of dimension 2 there are 3^2= 9 square sub-units of 1/3 the length. In the Koch flake, each side is repeatedly replaced by 4 = $3^{\log4/\log3}$ sides of length 1/3. Hence its fractal dimension is log4/log3 ~ 1.26.

The Nature of Chaos

Far from being the nemesis of order, or the primal ooze in which order is imposed, chaos is also the genesis of new form. Most complex systems arise from the mutual interaction between chaos and order, through bifurcation - ‘abrupt change of form under continuous underlying transformation. Bifurcation takes its name from ‘forking’ but really applies to all discrete transformation under continuous change. It is typified by the onset of opposing flight or fight reactions, and sudden transformations, such as a wave breaking, or a bubble bursting. Bifurcations can introduce new structure and hence increasing complexity, particularly in transition from chaos to order, in dynamics occurring at the ‘edge of chaos’.

Julia sets (white) of a complex logistic map (see below) are fractal regions on which the process is chaotic. On the insides (rainbow levels) and outside (rainbow shades) the process is ordered. The multiple basins inside converge towards attracting periodic points and the basin outside to infinity. Dynamics in the Julia sets are repelling, scrambled and sensitive to initial conditions, mapping any neighbourhood of the set over the whole. In every case, the dynamic is divided between complementary regions of order and chaos.



The failure to appreciate the generative nature of chaos has led to it being one of the last scientific frontiers to be discovered, over fifty years after relativity and quantum theory. This has happened because the human will to impose order, even among scientists, is so strong that somehow, in their rush to fit every phenomenon into a mechanistic world view, they ignored the fact that virtually all interesting natural phenomena involve chaos, from the waves on the beach, to the beauty of a forest, from our seemingly regular heartbeat to the patterns of our brain waves in the moment of 'eureka'!

Mathematicians distinguish dynamical chaos from a random, or stochastic process, in which critical events are determined by probabilities. Dynamical chaos is not simply disorder or randomness, but an internally unstable process. Chaotic systems may have well-defined dynamical formulations and may even be deterministic as classical systems, but this dynamic is one which doesn't settle down either into equilibrium or any particular periodicity or resonance, but wanders erratically over time in an unpredictable way which is deceptively similar to randomness.

Although chaotic systems may be precisely defined by a recursive formula or feedback process, they combine erratic behavior with long-term unpredictability which gives them just the character those seeking orderly prediction might fear. Chaotic bifurcations and a closely-related phenomenon called self-organized criticality are also frequently associated with crises such as cyclones, floods, avalanches, earthquakes and other catastrophic natural interventions.

The Lorenz butterfly effect: a puff of a butterfly's wing in a chaotic weather system can inflate into a tropical cyclone. Weather is thus intrinsically unpredictable, because the small differences in a tiny puff can grow and throw the whole system wildly off-course.

The essential characteristics or 'axioms' of classical chaos are threefold (Devaney R160):

1. Sensitive dependence: Lorenz, the father of chaos theory, was first to note the key characteristic of chaos in the 'butterfly effect', that the eddies of the wings of a butterfly flying in Hawaii could later become the seed of wild unpredictable fluctuation of a tropical cyclone hitting Fiji. This is 'sensitive dependence on initial conditions', in which arbitrarily small changes can later become amplified by a chaotic process or flow into global fluctuations.

2. Topological mixing: Any small open region will eventually become mixed over any other. This means the dynamics is very tangled, so any orbit goes almost everywhere in the 'phase space' of configurations of the system. This is precisely what happens in an egg-beater. This mixing property sometimes referred to as ergodicity makes the orbits or trajectories of a chaotic process appear random.

3. Dense periodicities: Chaotic dynamics is densely permeated with repelling periodic oscillations, often of infinitely many types, making for a great deal of hidden complexity.

Another way of encapsulating the latter two properties is to find a dense orbit – single trajectory in the system which comes arbitrarily close to every point in the space of states. These three combine to mean the dynamic is complex, unstable and unpredictable.

Sensitive dependence causes chaotic systems to eventually become fundamentally unpredictable even when they are deterministic. They cannot be accurately computed, since arbitrarily small errors in the computation rapidly escalate into global inaccuracies. This unpredictability is at the core of the difficulties of weather prediction and it also lies at the root of diverse phenomena, from the stock market, to the risk of nuclear holocaust.

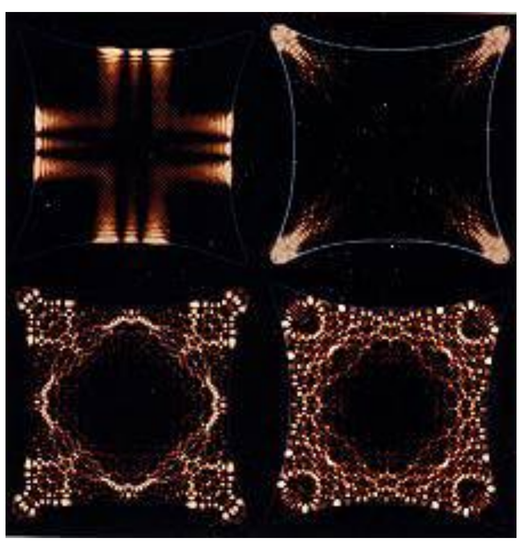
Associated with many chaotic systems and some statistical ones such as the 'drunkards walk' of Brownian motion, are beautiful, complex self-replicating patterns called fractals after their properties of self-similarity on smaller and smaller scales. Fractals are typified by the snowflake, trees, our lungs, the pattern of forest clearings and the biodiversity we associate with the evolutionary tree of life.

Whirls of water passing a stick (Schwenk R630).

Fractal processes do not have to be geometrically self-similar, but possess a self-replicating basis which leads to a non-integer dimensionality, hanging paradoxically between the integer values we are accustomed to. The Mandelbrot set, a portion of which is illustrated below has varying parts representing all the different ways we can make a feedback by squaring a complex number and adding a varying constant. The Koch flake fractal above replaces a side of a given length with 4 sides of length 1/3, thus having a dimension of log4/log3.

Chaos occurs throughout nature, from the fractal structures of galactic clusters, through the erratic orbits of comets to the dynamic many-body energetics of the atomic nucleus. It is manifest in biological systems from fractal tissues and organs such as the form of our lungs, from the seemingly regular heart beat, through brain dynamics, to the laser pulse.

Quantum systems modify the nature of chaos, suppressing some of the fine details of the interweaving of paths in the overlapping of waves possessed by all quanta, which tend to reveal the periodicities hidden in chaotic systems.



Wave functions, associated with the energy levels of a highly excited hydrogen atom in a strong magnetic field can exhibit both periodic and chaotic qualities. However the wave aspect smooths and partially suppresses chaos. The chaotic quantum wave functions are modified from the classical picture by 'scarring of the wave function' paradoxically concentrating the probability around hidden repelling periodic orbits (Gutzwiller R274).

Nevertheless molecular kinetics is a living example of quantum chaos - a form of unstable wave-particle billiards which affects all the processes and reactions occurring in our enzyme reactions.

Chaotic systems arise naturally from positive feedback processes because the positive feedback amplifies small differences, causing the instability we see in the butterfly effect. We shall see shortly that sexual selection is a potentially chaotic positive feedback process, prone to exponential runaway. In this respect it is complementary to the stabilizing ordered constraints imposed by natural selection.

Many apparently periodic phenomena are actually chaotic. The heart beat appears periodic, but the healthy heart is actually tuned by chaos. This allows the brain and heart pacemakers and the heart cells themselves all to keep in feedback resonance with one another and thus respond to changing circumstances. No two heartbeats have exactly the same interval between, but vary in a chaotic manner, similar to a dripping tap.

The universe is fractal in its manifestation in galactic clusters, galaxies, solar systems, stars planets and satellites and chaotic in its diversity. Some models of cosmic inflation and cosmic evolution are also fractal, however the supposedly chaotic formless cosmic origin in the 'big-bang' is described by symmetry-breaking of an almost symmetrical or isotropic germinal state, marred only by quantum fluctuation. Herein may lie the ultimate source of what we experience macroscopically as chaos - quantum uncertainty itself. Quantum uncertainty amplified by chaotic instability may be at the heart of other processes we describe as random, from molecular kinetics to tossing a coin.

Closely associated with chaotic and fractal processes are systems in a state of **self-organized criticality**. A sand pile always converges to a limiting angles where fractal avalanches maintain the entire system in a critically unstable state. Make the pile flatter and it heaps up again. Make it too steep and avalanches bring it back to criticality. Phenomena such as earthquakes, avalanches and neurons tuned to threshold excitation, share features of fractal instability. Hence smaller and larger earthquakes occur on a fractal pattern of frequencies.

Population Catastrophe and the Beauty of Fractals

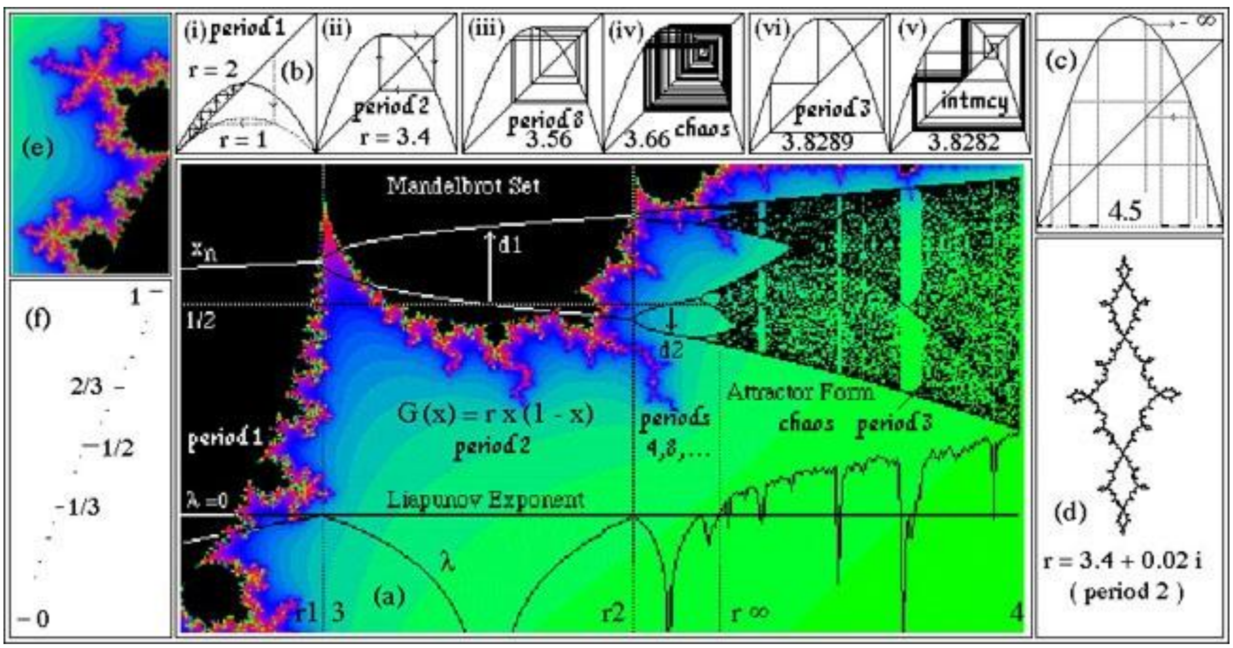
One of the most serious questions facing humanity is the boom or bust nature of human population explosion. Will we explode to an unsustainable population and wipe ourselves out with a global famine, or will we settle down nicely into equilibrium all a bit hungry but survivors as the population settles to a new plateau? The answer to this question is exceedingly complex and opens out the jewel of complexity hidden within chaos.

We will illustrate some of the elementary properties of chaos by looking at one of the simplest systems of all - an iterated feedback in one real variable. It is an archetype of the population explosion dilemma - how to feed a naturally growing population from a finite resource without boom and bust destroying the species.

The quadratic logistic map:

$$x_{n+1} = G(x_n) = rx_n(1-x_n)$$

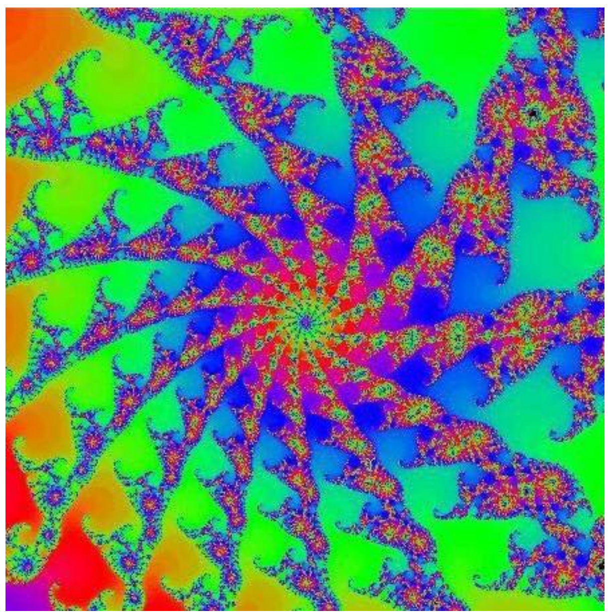
describes seasonal natural population growth given a constrained food supply. The term rxn gives natural growth by a factor r, while the additional term (1 - x_n) limits growth in proportion to the unconsumed food resource (e.g. the remaining area of arable land). Many possibilities arise, depending on the growth rate r. As the growth rate varies, the iteration goes through a sequence of different stages separated by sudden changes, or bifurcations. For small r the system tends to a fixed equilibrium point then becomes repelling bifurcating to form a flip-flop (period 2), subsequently period doubling to form periods 4, 8, 16 etc. through to infinity.



The logistic map: (a) As r increases from 2 to 4 the attracting final state is initially a single curve (an equilibrium point for each r) but then it repeatedly subdivides (pitchfork bifurcations) to make a rich and lean year (period 2) then periods 4, 8 etc., finally entering chaos (stippled bands). Subsequently there are windows of period 3, 5 etc. with abrupt transitions to and from chaos. The Lyapunov exponent indicates whether the system is chaotically amplifying small differences. During chaos it remains positive. The Mandelbrot set illustrates the fractal nature of the ordered and chaotic regimes when x and r are extended to the complex number plane. (b) A series of 2-D views of the iteration, including periods 1, 2 and 8 chaos, intermittency, and period 3. Pick an initial value x and find y by moving vertically to the curve $y = r\,x\,(1 - x)$. Next we let the new $x = y$ by moving horizontally to the sloping line. The two steps result in one iteration, i.e. $x_{n+1} = y = r\,x_n\,(1 - x_n)$. (c) The non-escaping points form a disconnected repelling fractal for r = 4.5. The attracting final set has now broken up, resulting in a fractal Julia set. (d) A connected Julia set for the complex logistic [x-axis vertical] plotted by inverse iteration taking all the 2^n square root solutions of the inverse function

$$x_{n-1} = \frac{1}{2}(1 \pm \sqrt{1 - 4(x_n/r)}).$$

At this point, a new erratic behaviour emerges, and the system wanders with no fixed period. chaos has appeared. All of the previous periodic attractors continue to exist hidden in the chaos as repellers, generating a tangled repelling flow, whose spreading causes sensitive dependence. As r increases further, windows of order, with new periods appear in a new and abrupt type of transition from chaos to order. There is yet a third type of chaotic transition represented by the mode-locked periodic feedback, as in the heart pacemaker and rotations on the periodic spirals of the Mandelbrot set illustrated below.

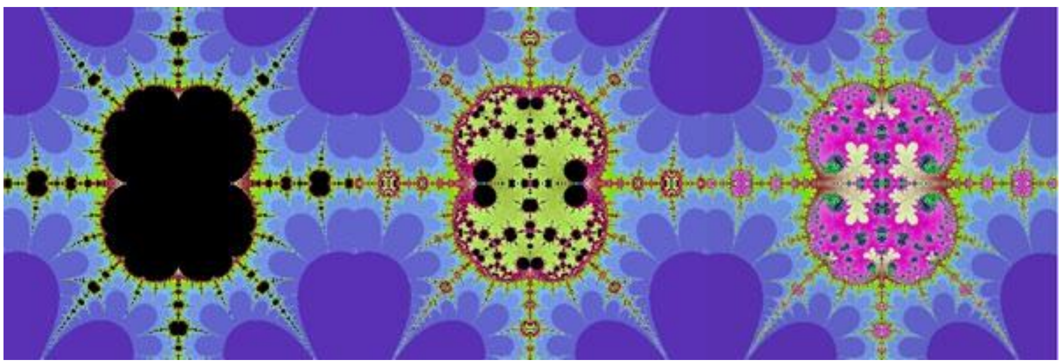


A portion of the Mandelbrot set of the logistic map. The fractal displays all possible quadratic dynamics. Although these vary and it is thus not exactly self-similar, it is nevertheless a fractal.

Finally a new situation emerges. The attractor becomes unstable. All that is left is a residual set of points, which do not escape, but are mapped chaotically among themselves. This Julia set has a complicated self-similar structure like a fern leaf or a snowflake and is called a fractal. Invariant sets of chaotic systems in several dimensions are frequently in the form of fractals, characterized by having a non-integer dimension (Peitgen et. al.R532, R533)

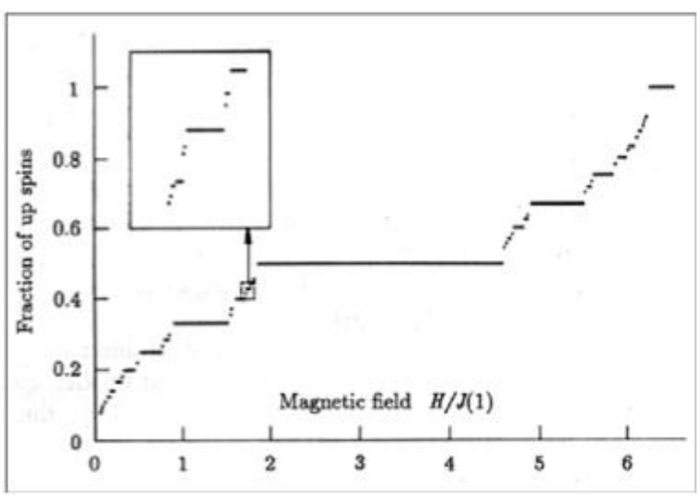
The fractal invariant sets can be seen in far more detail and beauty if the process is extended to the plane of complex numbers ($a + ib$ where i = -1). The Julia sets now clearly appear as fractals, each with a differing structure depending on the growth rate parameter.Â Â The Mandelbrot atlas set of all these dynamical variations has been dubbed the most complicated mathematical structure ever known.

Chaos occurs in a surprising variety of phenomena, many of which appear at the surface to be periodic. Both the heart beat and the dripping of a tap, although apparently periodic have chaotically intermittent variations in the beat period. The rings of Saturn and objects still remaining in the asteroid belt are governed by mode-locking chaos. Only those whose orbital periods have no rational (fractional) relationship remain, because all the fractionally-related orbits have long ago been thrown into the planets by a repeated sling-shot effect. When orbits of two astronomical bodies become mode-locked they interact strongly on a regular basis and the cumulative effect may throw the smaller one out of orbit. The asteroids remaining today are in a belt where the periods do not mode lock and have thus been left behind. More generally a large variety of systems from the weather through earthquakes, movement of the continental plates, chemical and electronic oscillations, secretion of enzymes, fluctuations in the stock market and collision of successive billiard balls, through to brain waves and possibly cognition itself, involve chaos or chaotic phases.



Julia set of the complex cosine pervades the plane, evolving firstly through an explosion to fractally fill the residual black region and then through an infinite set of fractal bifurcations.

Chaos presents us with new properties of nature which are connected with the development of complexity. A chaotic system contains within it a fractal structure with diverse dynamics, including a dense set of infinitely many periodicities.



Devil's fractal staircase for an anti-Ferromagnetic spin glass in a magnetic field (Schroeder R624).

Mode Locking and the Devil's Staircase

One particularly interesting feature of non-linear interaction manifest in many situations from the heartbeat to planetary orbits is the phenomenon of mode-locking.

Here coupled periodic phenomena with a periodicity close to a rational number become mode-locked into a rational relationship. This relationship is illustrated by the devilâ€™s staircase, a fractal function whose graph is continuous and increasing yet constant in a neighbourhood of every rational number. A close examination of the Mandelbrot set (p 383) will confirm that the bulbs on the set follow a devil's staircase pattern with the two largest bulbs above and below having 1/3 of a revolution (see the three dendrites) and the large bulb on the left corresponding to 1/2 a revolution, with the rest forming a series conforming to the devil's staircase arrangement, in numbers called a Farey Tree. In which any two fractions generate a descendent by the rule

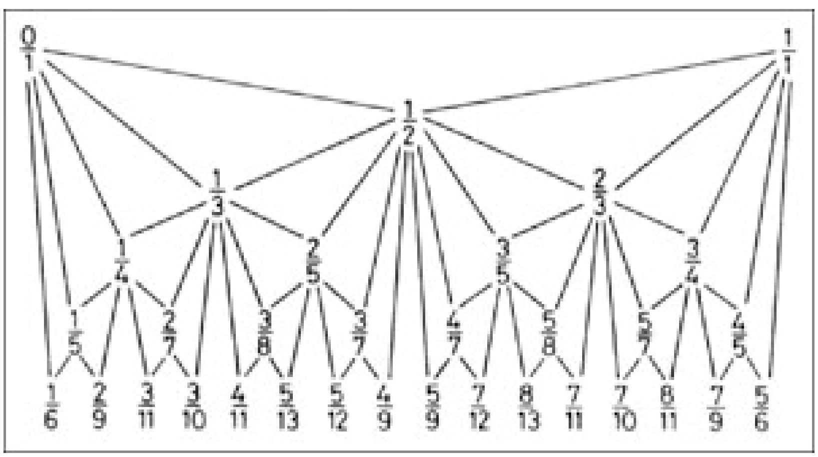
$$\frac{m_1}{n_1} \cdot \frac{m_2}{n_2} \rightarrow \frac{m_1 + m_2}{n_1 + n_2}$$

The Farey tree links all the mode-locked rationals in an infinite net. Starting from the top right if we alternate left and right we have a sequence of Fibonacci fractions converging to the golden mean. The Fibonacci numbers 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ... are generated by a similar relation

$$u = u_1 = 1, u_{n+1} = u_n + u_{n-1}$$

Their ratio tends in limit to $\gamma = 1 + \frac{1}{\gamma}$, giving $\gamma = \frac{1 \pm \sqrt{5}}{2} = 1.618, -0.618$,

the so-called golden mean number, which along with other such extreme irrationals, are the last numbers to become captured by mode-locked intervals.

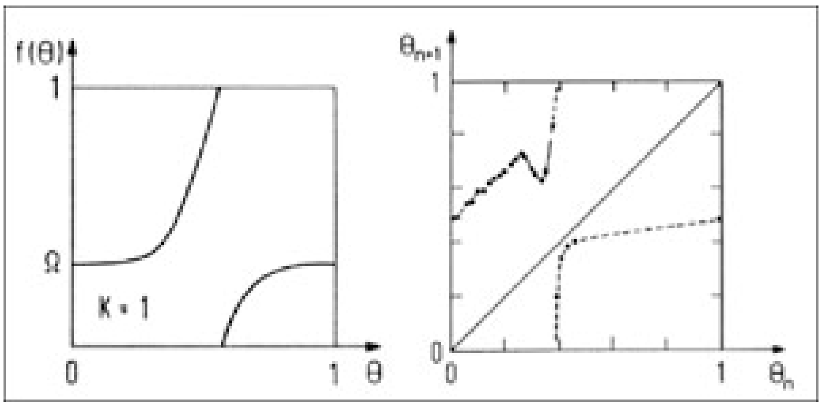


The Farey Tree

Mode-locking explains a vast variety of phenomena. The remaining asteroids all lie in the non-mode locked regions between Mars and Jupiter because rational orbital periods with respect to Jupiter have long ago been swept into the sun or planets by the pumping action mode-locking causes. Many apparently periodic phenomena such as the heart beat are also governed by mode-locking. A simple circular iteration called the circle map which can generate such behavior by modeling a sinusoidally-kicked rotator can be defined as follows:

$$\theta_{n+1} = f(\theta_n) = \theta_n + \Omega + K \sin\theta_n$$

where Ω is a constant twist by a fixed angle per iteration and the sine term causes a wobbling or pumping effect, resulting in an average twisting which may become mode-locked.

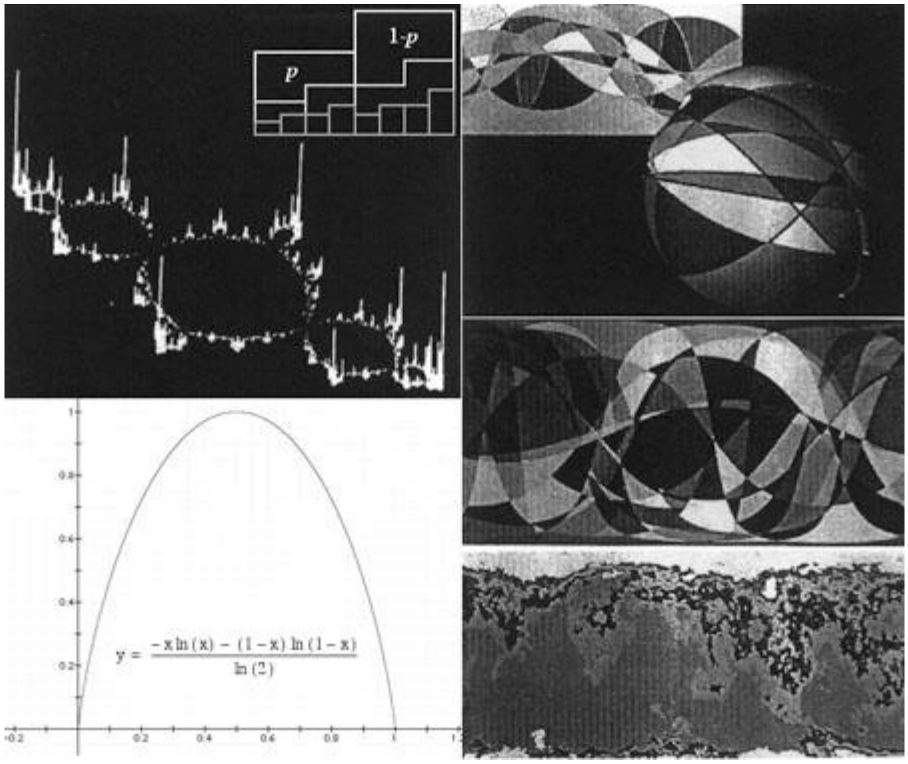


Left: The standard circle map. Right: An experimental setup using beating chicken heart cells (Schuster R628).

A 'spin glass' is considered to be a disordered system of spins just as a glass is a disordered quasi-crystalline substance. A ferromagnet has lowest energy when all its spins are aligned, giving a stable polarized lowest energy state. In an anti-ferromagnetic substance each adjacent pair of spins have a lowest energy state in which they are alternating up and down in a 3-D checkerboard pattern like the Na+ and Cl- ions in a crystal of salt. Closest spins have the most powerful neighbouring interaction with the strength falling off e.g. with the square of the distance. When there is no magnetic field the whole system is thus perfectly regular. However when an external field is applied, it becomes impossible for all the spins to resolve themselves in a single lowest energy arrangement, and many local minima become possible like a complex pattern of swamps and small lakes in a wild landscape. As shown on the previous page, this process is governed by mode-locking, implying that a spin glass contains fractal islands of partially resolved spins. A similar model may well describe human sexual mating patterns under a situation where one sex or the other is in scarcer supply.

Multifractality

Many fractal processes, from turbulence to the Julia sets of a quadratic mapping, distribute themselves unevenly, so that the process becomes fractally concentrated more in some regions than others. One can envisage this process if we cut an interval in half and give one side probability p and the other probability 1-p and repeat the process endlessly (see inset).



Top left: Julia set plotted by inverse iteration showing the multifractal distribution of how frequently differing subregions are visited. Inset fractal application of a probability in three stages. Inset an interval redistributed by probabilities p and 1-p iterates to a multifractal distribution. Lower left: The entropy function representing the information loss describes the multifractal spectrum of probabilities in the limit. Right: a continental landscape generated by repeated division of a planetary surface (see Peitgen R532, R533).

There are then fractal subpopulations corresponding to a given probability level, with a spectrum differing fractal dimensions hidden in the process. Many processes we see in nature are like this. For example if we divide the surface of the Earth into two broad regions and estimate that 60% of the population will be on one side and 40% on the other and repeat this process fractally, we will end up both with very sparse regions like the Sahara and very dense foci of population like New York. A similar process can generate continents.

The same process occurs with the inner dynamics of a Julia set as shown in the diagram if the iteration is run backwards to find all the 2n square root solutions n-steps back and so on. Some regions are visited a very large number of times, while others are visited exponentially rarely. This means it would take longer than the history of the universe to draw a Julia set if we simply used the raw inverse method as it stands.

Kitami National Park: Nature, from clouds, through the forms and patterns of vegetation, to the textures of rocks and the shapes of shorelines is an immensely complex system of overlapping fractals.

The Edge of Chaos and the Complexity of Nature

"Out of chaos comes order." Friedrich Nietzsche

A system which can bifurcate between chaos and order over time can enter a mixing phase of chaos and then retrieve structures hidden within chaos by bifurcating back into order. A chaotic system can likewise be tuned to display its hidden periodicities. Many types of system develop complex evolving structures in the transition region between order and chaos, sometimes called the 'edge of chaos'. The edge of chaos thus represents the region of sexual paradox between chaos and order where complexity becomes emergent.

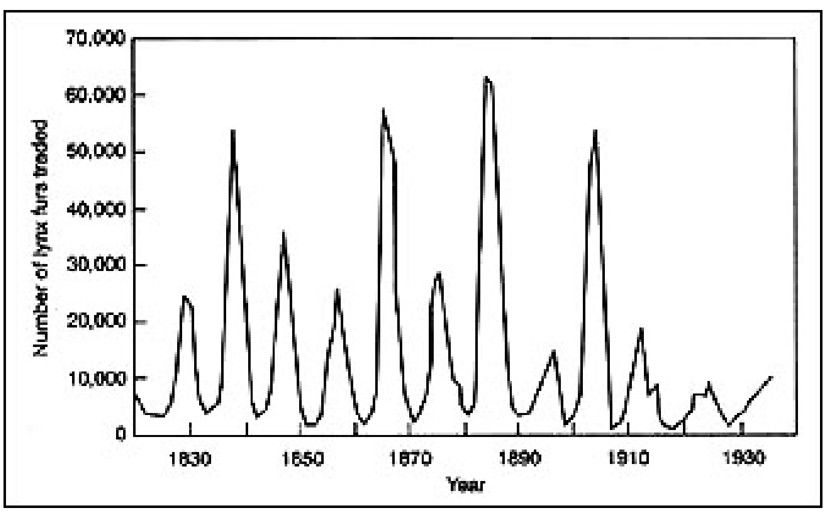
Nature and evolution are both described as complex systems evolving at the edge of chaos. Many of the most beautiful aspects of nature arise from their fractal structures and textures. Climax forests are chaotic systems, both in terms of their species diversity and their fluctuating population dynamics. Climax forest also displays a fractal dynamic which is central to its diversity. Natural disturbances from fire and flood, wind and storm damage, to large falling trees are fractal disturbances to which diverse species become adapted in disseminating seed in an ever more complex arrangement of species diversity. The forest is colonized in up to five strata from the top canopy to the floor each with their own ecosystemic complexity.

Both plants and animals are derived from fractal algorithms in nature and it is from these fractal algorithms that most of our understanding of form and diversity in nature comes. Evolution and its increasing complexity is a central instance of edge-of-chaos dynamics, as is our dynamical brain state, in both perception and problem-solving, especially when perceiving the chaotic diversity of nature itself for which we are highly adapted. It is the very sensitive dependence of chaos which ensures the brain remains completely adaptable to arbitrarily small differences.

The sunflower contains two sets of 34 and 55 spirals left and right enabling seed packing which avoids any periodic mode-locking. Many animal proportions also reflect the limiting ratio of two such magic numbers in the golden mean of 1:1.618, illustrating frozen chaos at work (see R556, R624).

An intriguing illustration of frozen chaos permeating biological organisms is the incidence of the golden mean as a ratio or angle in both animal and plant form. The twin spirals observed in plant forms, including the pineapple, pine cones, sunflowers and cacti occur at the golden mean angle 2p/g and generally have two related Fibonacci numbers. This prevents any ordered pattern of mode-locking which would prevent the seeds of the sunflower packing together properly.

Similarly many human proportions, from successive digit bones, the relative distance from the navel to the head and feet, the widths of successive incisors and the nose, mouth and eyes all conform to the golden mean. This is the last, most irrational number to submit to mode locking, as do the orbits of the remaining asteroids in relation to the orbit of Jupiter. Mode locking can also bee seen in the 13 arms of the Mandelbrot portion above, where the dynamic is making 1/13 of a revolution.



The lynx is a species with regularly, yet erratically oscillating numbers. It was once believed that lynxes were partners in a dynamically unstable association with their main prey, the snowshoe hare. Recently it has been recognized that the cycle is driven by the interaction between hares and their food plants, with the lynxes being carried along more or less passively by changes in the abundance of hares (Leakey R316).

In addition to this, the potentially chaotic population dynamics we have seen in the logistic function is displayed in many natural populations making population dynamics unstable from season to season and sensitively dependent on changes in the environment. For this reason, we have to be very careful when considering the major impacts we are making on natural ecosystems, lest chaos and bifurcation compound the problems we initiate.

It is important to note that population dynamics may cause paradoxical situations to arise. For example we usually think of a predator-prey relationship as exploitative. However a predator acts to reduce the growth rate of a population and thus protects it from boom and bust population crisis in which the prey multiplies so fast that it eats all the available food and dies en masse through starvation. Thus predator and prey are caught in a kind of prisoners dilemma relationship which is both destructive and protective at the same time.

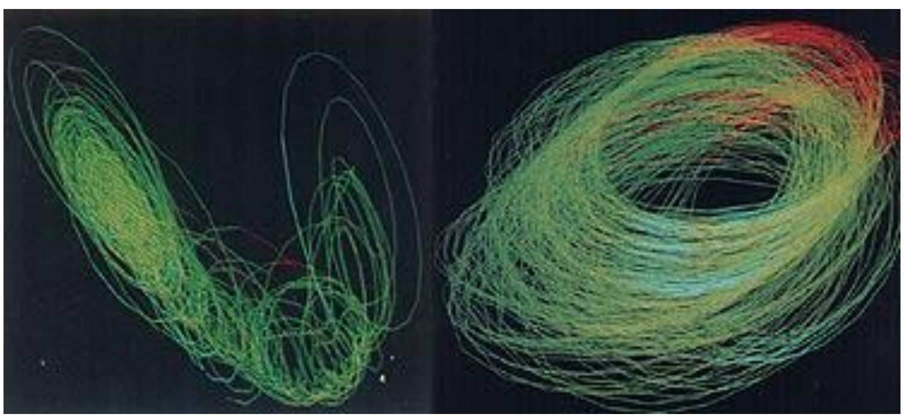
Similar considerations apply to parasites and hosts. A central development of this dynamical relationship we shall see next is the idea that a prisoners dilemma genetic ‘arms race’ between parasites and hosts led to sexual evolution to promote genetic variety and hence resistance to disease. This mutual adaption arms race thus required each of the competing organisms to become capable of sexual recombination to survive the others changes.

Once sexuality became established, sexual selection began to become a fundamental driving force complementary to natural selection. Because natural selection tends to operate as environmental or inter-species constraints on survival it is both stable and predominantly a negative feedback. In addition the vast majority of mutations are deleterious.

The peacock's tail illustrates how sexual selection can become a runaway positive feedback process, leading to chaotic unpredictability. Once again we see hints of Fibonacci golden mean spirals.

Sexual selection has very different characteristics from natural selection. Firstly it acts not negatively on survival but positively on reproduction. It is also an iterative feedback process with strong positive feedback characteristics. Female reproductive choice acts as a capricious and variable positive feedback which, as it adapts to competing display by becoming more discerning, drives male evolution into potential runaway. Mutual mate selection can also have powerful effects. This leads to sexual selection becoming a potentially chaotic positive feedback force complementing the stabilizing effects of natural selection. These effects are again complemented by the opposing effects of mutations and recombination as genetic modifiers held in check by selection retaining only the viable options.

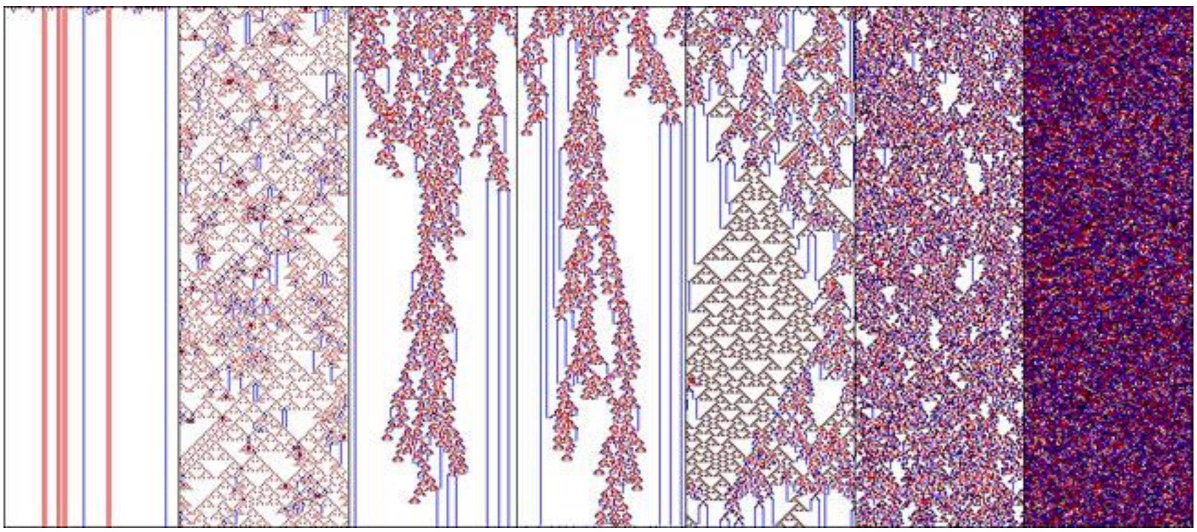
These effects result in a deep connection between sexual paradox and edge of chaos complexity. Broadly speaking the condition of sexual paradox induces sensitively unstable dynamics which lead to complex systems dynamics at the edge of chaos because the actions of each of the partially opposing forces are frustrated from imposing order. Loss of sexual paradox leads to degeneracy, with a dominant stable process and consequently reduced complexity and reduced viability. Thus maintaining sexual paradox in evolution and climax diversity in planetary abundance and resilience go hand in hand. Although our gatherer-hunter origins appear to be sexually paradoxical, many aspects of human culture show loss of sexual paradox into degeneracies of patriarchal sexual and natural dominion involving boom and bust and rape of the planet's diversity. These are accompanied by very worrying instances of loss of complexity which need urgent correction to ensure human viability.



Strange attractors in brain dynamics Freeman (R226)

A final example of the interdependence of chaos and order in the development of complexity is illustrated in the brain, (p 323), which is not simply a digital computer but displays prominent dynamical behavior, illustrated in the broad spectrum waves of excitation in the electroencephalogram. This excitation is distributed across the cortex in a manner consistent with parallel distributed processing. Both perception and cognition can be modeled as a transition from a state of chaos representing the unrecognized condition, or the unsolved problem, to a state of order. This process can be modeled as a transition: from high energy chaos, 'exploring' its internal space without getting stuck in any 'rut'; to order, as the energy is reduced so as to flow towards a minimum, through the capture of the system by a learned attractor in recognition, or the bifurcation of the system to form a new attractor. An insight 'eureka' often happens instantaneously, from a state of relative confusion, indicating a single transition from chaos to new order representing the 'knowing' state. The chaotic state is thus the progenitor of new order, rather than mere manipulation of order itself. Rather the order imposed by the problem becomes a boundary condition for chaotic resolution.

Cellular Automata and Chaos



A series of 1-dimensional cellular automata with l varying from order (0.23) through complexity at the edge of chaos (0.33) to deep chaos (0.86). All complex states in 3 eventually expire, but not in 4.

Several of these attributes of order, chaos and complexity can be displayed in some the simplest digital feedback processes, of all called cellular automata (see Wolfram R606). These contain a formula expressing the successive states in a grid of cells in terms of their immediate neighbours. Depending on the nature of the formula, and the degree of overlap, such systems can be defined to involve ordered equilibrium and periodicity, chaotic mixing of states, or complexity. Systems close to the edge between order and chaos display the capacity for increasing complexity.

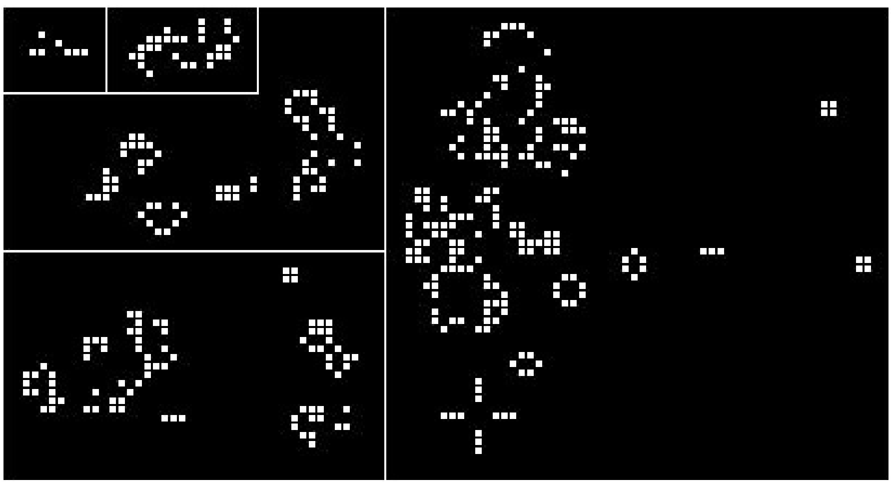
For example if we consider the simplest 1-dimensional cellular automaton which has only values 0 and 1 in a given cell and determines the new state of any cell from the three immediate predecessors in the preceeding row, we can define a unique rule as a number.

There are 23=8 different arrangements of a cells three immediate predecessors. To specify the rule we thus need to specify a 0 or 1 in each position giving 28 = rules numbering 0 to 255.

Rule 110, for example, displays edge-of-chaos complexity. This system is even capable of functioning as a universal computer, because various sub-states can exchange information and perform the fundamental computations of symbolic logic.

Rule 30 displays the key features of chaos in digital form. The eventual states are sensitively dependent on the initial conditions. If we put a Cantor topology on the space of configurations, we find periodic solutions are dense (the is a finite configuration with any finite pattern of cells) and any two states become mixed in the sense that a configuration containing one pattern leads to one containing the other. Finally there is a dense orbit, a configuration that eventually displays any given finite pattern of cells.

A famous example of complexity with universal computation is Conway's 'game of life' in which a given cell survives if two or three neighbours out of the possible eight are alive and is 'born' if precisely three are alive. The 'game of life' behaves in a similar manner to a complex dynamical system at the edge of chaos. Here successive states show increasing complexity, including drifters capable of logical computation. Such processes, including 2-D cellular automata simulations of the prisoners' dilemma, (p 21), may thus become formally undecidable because of the Turing halting problem (p 351). Conway's game of life is equivalent to a prisoners' dilemma game where cooperation is incited by three cooperating neighbours and the status quo maintained by two, with other values leading to defection.



A sequence of states in the game of life

Below are illustrated some variants of the game of life in which a variety of slightly higher rewards are given for cooperation, leading to permanent equilibrium between cooperators and defectors similar to the results of Nowakâ€™s more detailed simulations, (p 21) but using only simple rules of the same type as Conwayâ€™s.

Unlike the game of life, consciousness is not bound to a discrete classical logic. Ultimately, through chaotic sensitivity, the conscious brain may be able to access the quantum realm and putative forms of quantum computing and transactional space-time hand-shaking, manifestations of the weird properties of uncertainty, non-locality and entanglement, arising from quantum complementarity between wave and particle aspects. Consciousness appears to use these deeper complementarities within quantum chaos to anticipate potentially incomputable complexities and to affect physical outcomes through the application of conscious will. Here we come to the deepest expression of that complementarity in logical and existential paradox of which chaos and order are also a reflection. This is where sexual paradox enters its quintessence.

2-D cellular automaton variants of the game of life. If Conway's game is represented as 001300000 meaning death for 0, 1 and 4-8 live neighbours, the status quo for 2 and birth for 3. From top left (a) 001330000 showing long-term equilibrium between defectors (yellow and orange) and cooperators (black and blue). (b) Symmetrical fractal states generated by a single defection for 001300001- very close to the game of life - giving an expanding regime of computational unpredictability for the continuity of life (cooperation) amid defection (death), similar to Conway's game. (c) 301300001, (d) 301330001 and (e) 331300001, in which rare cooperators are rewarded.